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METRICAL PROPERTIES OF NESTED PARTITIONS FOR IMAGE RETRIEVAL

P. Grimm[1], Ye. Kinoshenko[2], S. Mashtalir[3], V. Shlyahov[3]

[1] University of Applied Sciences and Arts, Germany
[2] Kharkiv Medical Academy of Postgraduate Education, Ukraine
[3] Kharkiv National University of Radio Electronics, Lenin Ave., 14, 61166, Kharkiv, Ukraine

ABSTRACT

Image processing for the efficient retrieval should perform the ability of data granulation and interpretation. In this paper the properties of metric on nested partitions which allow to analyze objects represented at different levels of granularity and abstraction is considered. It also ensures a retrieval of image parts corresponding to the searched objects i.e. provides a search criterion for background independent objects.

Index Terms – Image retrieval, metrics, data granulation

1. INTRODUCTION

Image processing and interpretation is an important task in various applications of sensor technologies and communication systems. For this an arbitrary matching involves quantifiable similarities. A match is not merely a correspondence but a conformity that has been quantified according to its 'goodness'. Therefore, this measure of goodness must be the matching metric at best since it gives additional opportunities e.g. to retrieve images with search speedup or to find totally correct and complete segmentation when elements of obtained partitions uniquely correspond with objects in the input image. Thus, there arise many reasons to study metrical properties of nested set partitions for solving a semantic gap between low-level features and high-level human concept since partitions are merit models of arbitrary strong hierarchical clustering.

Validation of image semantic truthful representation is a base of a reasonable compromise between under- and over-segmentation what constitutes a major cause of image understanding and recognition complexity. Desirable image simplification level has to provide the explosion of image content. In addition, under clustering of image collections we have to analyze partitions (probably nested) to get image classes with given or determined similarity threshold. Therefore, we have to know: what separates the levels of detailing? what ties these levels together? how do the levels interact with each other? In this regard, a problem of impartial partition matching is of great interest. Among the most promising metrics which particularly have desirable properties one can name the Earth Mover's Distance [1], Meyla variation of information [2], Mirkin metric [3], van Dongen metric [4] and metric on partitions of arbitrary measurable sets [5,6]. Although all these metrics have their specifics and all succeed to solve various tasks of measuring the granulated visual information there are still many requirements for the image recognition application which arise the need to have a tool for managing the refining or roughening of granulations specified by quotient sets. Possibility to operate with image as a dynamic hierarchical structure will allow to solve many tasks of image processing including image retrieval even object detection more effectively and also approach to many new problems on a higher intelligence level. For arbitrary measurable sets in [6] the metric with such measure type as length, area, volume, mass distribution, probability distribution, cardinality, etc. has been introduced. This metric is based on cumulative evaluation of resemblance and dissimilarity. The approach allows to get more precise definition of partitions distances using nesting quotient set ordering with respect to inclusion solely.

In the paper we have introduced various forms of metric on nested partitions and investigated geometrical properties relatively offered metric.

2. NESTED PARTITIONS METRICS BACKGROUND

Let some finite set Ω with measure $\mu(\circ)$ characterise information which should be granulized in the results of analysis. Talking about measure we understand the number of points (in the finite case), length, square, volume, mass distribution, etc. We shall assume that measure is finite, i.e. $\mu(\Omega) < \infty$ and F_Ω is a set of all finite subsets of the set Ω . We shall consider Π_Ω set of all finite (in terms of the elements number) partitions of the set Ω , i.e. $\alpha \in \Pi_\Omega \Leftrightarrow \alpha = \{A_i\}_{i=1}^n$

$$\begin{cases} A_i \in F_\Omega \quad \forall i \in \{1,2,\ldots,n\}; \\ A_i \cap A_{i'} = \varnothing, \ \ \forall i \neq i' \in \{1,2,\ldots,n\}; \\ A_1 \cup \ldots \cup A_n = \Omega \, . \end{cases} \quad (1)$$

Let some arbitrary partitions $\alpha, \beta, \gamma \in \Pi_\Omega$ ($\alpha = \{A_i\}_{i=1}^n, \beta = \{B_j\}_{j=1}^m, \gamma = \{C_k\}_{k=1}^l$) be given.

The task is to define, research and interpret metrical properties under different relations between α, β, γ.

We shall explore set Π_Ω of finite partitions of arbitrary measurable set Ω.

Earlier in [5] a metric was defined and researched

$$\rho(\alpha,\beta) = \sum_{i=1}^{n} \mu(A_i \triangle B_j)\mu(A_i \cap B_j), \qquad (2)$$

where $\triangle$ is a symmetrical difference of the sets.

Let us show that functional (2) has the equivalent form

$$\rho(\alpha, \beta) = \sum_{i=1}^{n} [\mu(A_i)]^2 + \sum_{j=1}^{m} [\mu(B_j)]^2 - 2\sum_{i=1}^{n}\sum_{j=1}^{m} [\mu(A_i \cap B_j)]^2, \qquad (3)$$

which practically coincides with the Mirkin metric [3].

For any measurable (finite) sets E and F takes place

$$\mu(E \triangle F) = \mu(E) + \mu(F) - 2\mu(E \cap F). \qquad (4)$$

Indeed, taking into account equalities $E = (E \setminus F) \cup (E \cap F)$ and $F = (F \setminus E) \cup (F \cap E)$, at that $E \setminus F$ and $E \cap F$ do not intersect, from the measure's property of additivity we get

$$\mu(E) = \mu(E \setminus F) + \mu(F \cap E),$$

$$\mu(F) = \mu(F \setminus E) + \mu(E \cap F),$$

summarizing which with consideration of $E \triangle F = (E \setminus F) \cup (F \setminus E)$, we get

$$\mu(E) + \mu(F) = \mu(E \setminus F) + \mu(F \setminus E) + 2\mu(E \cap F) =$$

$$= \mu(E \triangle F) + 2\mu(E \cap F)$$

what is equivalent to (4).

Applying (4) to A_i and B_j, we shall represent (2) in the form of

$$\rho(\alpha,\beta) = \sum_{i=1}^{n}\sum_{j=1}^{m} \mu(A_i \triangle B_j)\mu(A_i \cap B_j) =$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{m} \mu(A_i \cap B_j)[\mu(A_i) + \mu(B_j) - 2\mu(A_i \cap B_j)] =$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{m} \mu(A_i)\mu(A_i \cap B_j) + \sum_{i=1}^{n}\sum_{j=1}^{m} \mu(B_j)\mu(A_i \cap B_j) -$$

$$-2\sum_{i=1}^{n}\sum_{j=1}^{m} [\mu(A_i \cap B_j)]^2 = \sum_{i=1}^{n} \mu(A_i) \sum_{j=1}^{m} \mu(A_i \cap B_j) +$$

$$+ \sum_{j=1}^{m} \mu(B_j) \sum_{i=1}^{n} \mu(B_j \cap A_i) - 2\sum_{i=1}^{n}\sum_{j=1}^{m} [\mu(A_i \cap B_j)]^2.$$

In the chain of these equations we shall extract sums of type

$$\sum_{j=1}^{m} \mu(A_i \cap B_j), \ \sum_{i=1}^{n} \mu(B_j \cap A_i),$$

and, applying to them property $\forall D \subseteq \Omega$, $\forall \alpha \in \Pi_\Omega$ $\mu(D) = \sum_{i=1}^{n} \mu(D \cap A_i)$, which directly follows from measure's additivity, implying under D A_i and B_j correspondingly, we shall get

$$\sum_{j=1}^{m} \mu(A_i \cap B_j) = \mu(A_i), \ \sum_{i=1}^{n} \mu(B_j \cap A_i) = \mu(B_j)$$

for any i and j, where $i = \overline{1, n}$, $j = \overline{1, m}$, what after substitution to (5) gives us (2).

For the research of properties of the embedded partitions we shall introduce two interrelated functionals

$$S(\alpha) = \sum_{i=1}^{n} \mu^2(A_i), \ G(\alpha) = \rho(\alpha, O),$$

where O is a conditionally 'zero' partition, which consist of the initial set Ω itself Ω, i.e. $O = \{\Omega\}$.

The metric on partitions is conveniently defined through these functionals, and its properties consist of the properties of these functionals. Further we assume that given below relations for $\rho(\alpha,\beta)$, $S(\alpha)$ and $G(\alpha)$ are fulfilled for any finite partitions (if some limitations are needed, they will be especially noted). It should be emphasised that under $\alpha\beta$ we shall understand the intersection of two arbitrary partitions.

More precisely: intersection $\alpha\beta = \{A_i \cap B_j\}_{\substack{i=\overline{1,n} \\ j=\overline{1,m}}}$ of pair $\alpha, \beta \in \Pi_\Omega$ of arbitrary partitions is also a partition.

Indeed, if to take an arbitrary element $\omega \in \Omega$, then as α and β are partitions, the numbers $j \in \{1, \ldots, m\}$ can be found, for which $\omega \in A_i$ and $\omega \in B_j$, i.e. $\omega \in A_i \cap B_j$. Thus,

$$\bigcup_{i=1}^{n} \bigcup_{j=1}^{m} \{A_i \cap B_j\} = \Omega .$$

On the other hand under associativity and commutativity of the sets intersection operation, we can record

$$(A_i \cap B_j) \cap (A_{i'} \cap B_{j'}) = (A_i \cap A_{i'}) \cap (B_{j'} \cap B_j).$$

But since pairs (i, j) and (i', j') are not equal, then either $i \neq i'$, or $j \neq j'$, or these equations are fulfilled simultaneously, what means that one of the sets $A_i \cap A_{i'}$ or $B_{j'} \cap B_j$ is equal to $\varnothing$, as they belong to partitions α and β correspondingly. Thus in result we have $(A_i \cap B_j) \cap (A_{i'} \cap B_{j'}) = \varnothing$ for any pairs $(i, j), (i', j') \in \{1, \ldots, n\} \times \{1, \ldots, m\}$, i.e. together with equality (7) we get fulfilment of the relations (1), what was required.

3. METRICAL PROPERTIES

We shall stop at the main properties $\rho(\alpha,\beta)$, $S(\alpha)$ and $G(\alpha)$.

P r o p e r t y 1. $\rho(\alpha,\beta) = S(\alpha) + S(\beta) - 2S(\alpha\beta)$.

P r o p e r t y 2. $\rho(\alpha,\beta) = 2G(\alpha\beta) - G(\alpha) - G(\beta)$.

These properties follow from the definition of the initial metric and functionals. The results of these definitions are properties 3 and 4, which point at the connection between $S(\alpha)$, $G(\alpha)$ and valuation of the functionals on the partition $\mathbb{O}$.

P r o p e r t y 3. $S(\mathbb{O}) = \mu^2(\Omega),\ G(\mathbb{O}) = \mathbb{O}$.

P r o p e r t y 4.

i) $G(\alpha) = S(\mathbb{O}) - S(\alpha) = \mu^2(\Omega) - S(\alpha)$;

ii) $S(\alpha) = S(\mathbb{O}) - G(\alpha) = \mu^2(\Omega) - G(\alpha)$;

iii) $S(\alpha) + G(\alpha) = S(\mathbb{O}) = \mu^2(\Omega)$.

From triangular inequality there follow two more properties for three partitions.

P r o p e r t y 5. $S(\alpha) + S(\beta\gamma) \geq S(\alpha\beta) + S(\alpha\gamma)$.

P r o p e r t y 6. $G(\alpha\beta) + G(\alpha\gamma) \geq G(\alpha) + G(\beta\gamma)$.

From the positiveness of the metric we get the next properties:

P r o p e r t y 7.
$2S(\alpha\beta) \leq S(\alpha) + S(\beta) \leq S(\alpha\beta) + \mu^2(\Omega)$.

P r o p e r t y 8.
$2G(\alpha\beta) \geq G(\alpha) + G(\beta) \geq G(\alpha\beta) + \mu^2(\Omega)$.

Further for the briefness we will list the properties of functional $S(\alpha)$. Properties of the metric and functional $G(\alpha)$ can be easily obtained, as hereinabove their direct relation was specified.
The stated before properties result in the next property:

P r o p e r t y 9. $S(\alpha\beta) < \mu^2(\Omega)$.

P r o p e r t y 10. $S(\alpha) < \mu^2(\Omega)$.

P r o p e r t y 11. $S(\alpha\beta) \leq S(\alpha)$.

P r o p e r t y 12.
If $\alpha,\beta \subset \gamma$, then $\rho(\alpha,\beta) + S(\alpha\beta) \leq S(\gamma)$.

P r o p e r t y 13.
If $\alpha,\beta \subset \gamma$, then $S(\alpha\beta) < S(\gamma),\ \rho(\alpha,\beta) < S(\gamma)$.

P r o p e r t y 14. $\rho(\alpha,\beta) < \mu^2(\Omega)$.

P r o p e r t y 15.

Π_Ω is an open set, bounded (limited) but does not include its border.

P r o p e r t y 16. If $\alpha \subset \beta$, then

i) $\rho(\alpha,\beta) = S(\beta) - S(\alpha) = G(\alpha) - G(\beta)$;

ii) $G(\alpha) \geq G(\beta)$, $S(\alpha) \leq S(\beta)$;

iii) elements $\Pi_\Omega : \mathbb{O}$, α, β conditionally lay on one 'line' in the sense of

$$\rho(\mathbb{O},\alpha) = \rho(\mathbb{O},\beta) + \rho(\alpha,\beta),$$

i.e. β is situated between $\mathbb{O}$ and α.

P r o p e r t y 17.

Under 'detalization' of partition α functionals $S(\alpha)$ and $G(\alpha)$ are behaving as following: $S(\alpha)$ is decreasing, $G(\alpha)$ is increasing.

P r o p e r t y 18. The case when

$$\lim_{n\to\infty} \max \mu(A_i) = 0,$$

when $\alpha = \{A_1, \ldots, A_n\}$, $i \in \overline{1,n}$, then

$$\lim_{n\to\infty} G(\alpha) = \mu^2(\Omega), \quad \lim_{n\to\infty} S(\alpha) = 0.$$

P r o p e r t y 19.

If $\alpha \subset \beta \subset \gamma$, then $\rho(\alpha,\beta) + S(\alpha\beta) \leq S(\gamma)$.

P r o p e r t y 20.

If $\alpha \subset \beta \subset \gamma$, then $\rho(\alpha,\beta) + \rho(\beta,\gamma) = \rho(\alpha,\gamma)$,

means that 'point' β lays on the 'line', passing through 'points' α and γ between them.

P r o p e r t y 21.

$$\forall \alpha, \beta, \gamma \in \Pi_\Omega \Rightarrow S(\gamma) \geq S(\alpha\gamma) + S(\beta\gamma) - S(\alpha\beta).$$

Indeed, from Property 1 and triangular inequality we get

$$S(\alpha) + S(\gamma) - 2S(\alpha\gamma) + S(\beta) + S(\gamma) - 2S(\beta\gamma) \geq$$
$$\geq S(\alpha) + S(\beta) - 2S(\alpha\beta),$$

whence it follows the required.

In conclusion we shall discuss 'geometry' of set Π_Ω.

1. Π_Ω belongs to a circle with centre in $\mathbb{O}$ and radius $\mu^2(\Omega)$.
2. Π_Ω has 'diameter' $\mu^2(\Omega)$, as $\rho(\alpha,\beta) < \mu^2(\Omega)$.
3. Π_Ω is a sheaf of 'lines', which pass through $\mathbb{O}$, but do not tend to infinity under the limitation.
4. Π_Ω contains lines which intersect infinite times.

Let us refine this statement. If we start from any 'point' α, on moving along any 'line', it would mean the detalization process of partition α. At that if we consider two different kind of 'detalization' in form of partitions β_1 and β_2, then 'lines', passing through 'points' α and β_1, and also α and β_2, will be 'intersecting' in point $\beta_1\beta_2$ and so on. It can be illustrated as following.

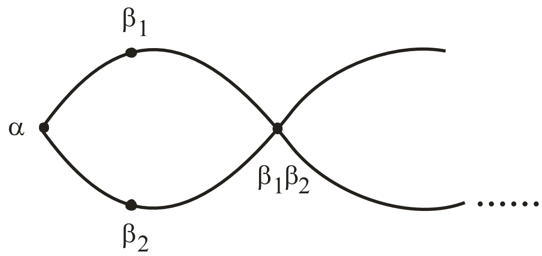

Figure. 1 The sheaf of infinitely intersecting lines

We can conditionally singe out some 'infinite' partition (as the limit of the infinite detailing of the $\mathbb{O}$ partition), which is embedded into any other one. Then all 'lines' from Π_Ω can start from any other 'point' α, but will always include 'point' $\mathbb{O}$.

4. EXPERIMENTS AND DISCUSSION

The explicit metrics expressions provide formulations and proofs of geometric properties of nested partitions. On the whole, offered metrics on partial ordered quotient sets are underlying tools of image content analysis as they comply with construction of a hierarchy either the top-down or the bottom-up approaches and one can explain wholes by decomposing them into smaller and smaller parts or alternatively one can construct wholes from smaller parts.

Efficient matching the collections of image partitions stipulated by the properties of the metric on nested partitions does not only provide the similarity measure for image segmentations, which enables the use of the image retrieval systems in a wider variety of applications but it also solves the problem of dependency on the segmentation results when obtaining partitions with low level of detail and high level of detail (Figure2).

Figure. 2 Image and its partitions with different level of detail.

Experiments have shown that as background is the biggest "object" in the image metric of partitions, we can search for images with the same background layout. In many situations only a part of the image which corresponds to object(s) need to be retrieved. Here a search criterion for background independent objects comes can be suggested.

From the image understanding point of view, the method allows to analyze objects represented at different levels of granularity and abstraction.

Figure. 3 The result of most similar in sense of nesting criterion images retrieved from database for each of partitions of the query image (Fig. 2)

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